3.2.2 Rational Method

One of the most commonly used equations for the calculation of peak flow from small areas is the Rational formula, given as:

$$Q = (CIA)/K_u \tag{3-1}$$

where:

Q	=	Flow, m^3/s (ft^3/s)
C	=	Dimensionless runoff coefficient
I	=	Rainfall intensity, mm/hr (in/hr)
A	=	Drainage area, hectares, ha (acres)
K_u	=	Units conversion factor equal to 360 (1.0 in English Units)

Assumptions inherent in the Rational formula are as follows:[6]

- Peak flow occurs when the entire watershed is contributing to the flow.
- Rainfall intensity is the same over the entire drainage area.
- Rainfall intensity is uniform over a time duration equal to the time of concentration, t_c. The time of concentration is the time required for water to travel from the hydraulically most remote point of the basin to the point of interest.
- Frequency of the computed peak flow is the same as that of the rainfall intensity, i.e., the 10-year rainfall intensity is assumed to produce the 10-year peak flow.
- Coefficient of runoff is the same for all storms of all recurrence probabilities.

Because of these inherent assumptions, the Rational formula should only be applied to drainage areas smaller than 80 ha (200 ac).[8]

3.2.2.1 Runoff Coefficient

The runoff coefficient, C, in Equation 3-1 is a function of the ground cover and a host of other hydrologic abstractions. It relates the estimated peak discharge to a theoretical maximum of 100% runoff. Typical values for C are given in Table 3-1. If the basin contains varying amounts of different land cover or other abstractions, a composite coefficient can be calculated through areal weighing as follows:[6]

$$\text{Weighted } C = \Sigma (C_x A_x)/A_{total} \tag{3-2}$$

where:

x = Subscript designating values for incremental areas with consistent land cover

Table 3-1. Runoff Coefficients for Rational Formula.[14]

Type of Drainage Area	Runoff Coefficient, C*
Business:	
Downtown areas	0.70 - 0.95
Neighborhood areas	0.50 - 0.70
Residential:	
Single-family areas	0.30 - 0.50
Multi-units, detached	0.40 - 0.60
Multi-units, attached	0.60 - 0.75
Suburban	0.25 - 0.40
Apartment dwelling areas	0.50 - 0.70
Industrial:	
Light areas	0.50 - 0.80
Heavy areas	0.60 - 0.90
Parks, cemeteries	0.10 - 0.25
Playgrounds	0.20 - 0.40
Railroad yard areas	0.20 - 0.40
Unimproved areas	0.10 - 0.30
Lawns:	
Sandy soil, flat, 2%	0.05 - 0.10
Sandy soil, average, 2 - 7%	0.10 - 0.15
Sandy soil, steep, 7%	0.15 - 0.20
Heavy soil, flat, 2%	0.13 - 0.17
Heavy soil, average, 2 - 7%	0.18 - 0.22
Heavy soil, steep, 7%	0.25 - 0.35
Streets:	
Asphaltic	0.70 - 0.95
Concrete	0.80 - 0.95
Brick	0.70 - 0.85
Drives and walks	0.75 - 0.85
Roofs	0.75 - 0.95

*Higher values are usually appropriate for steeply sloped areas and longer return periods because infiltration and other losses have a proportionally smaller effect on runoff in these cases.

The following example illustrates the calculation of the runoff coefficient, C, using area weighing.

Example 3-1

Given: The following existing and proposed land uses:

Existing conditions (unimproved):

Land Use	*Area, ha*	*(ac)*	*Runoff Coefficient, C*
Unimproved Grass	*8.95*	*(22.1)*	*0.25*
Grass	*8.60*	*(21.2)*	*0.22*
Total =	*17.55*	*(43.3)*	

Proposed conditions (improved):

Land Use	*Area, ha*	*(ac)*	*Runoff Coefficient, C*
Paved	*2.20*	*(5.4)*	*0.90*
Lawn	*0.66*	*(1.6)*	*0.15*
Unimproved Grass	*7.52*	*(18.6)*	*0.25*
Grass	*7.17*	*(17.7)*	*0.22*
Total =	*17.55*	*(43.3)*	

Find: *Weighted runoff coefficient, C, for existing and proposed conditions.*

Solution:

SI Units

Step 1: Determine Weighted C for existing (unimproved) conditions using Equation 3-2.

$$\text{Weighted } C = \sum (C_x A_x)/A$$
$$=[(8.95)(0.25) + (8.60)(0.22)] / (17.55)$$
$$\text{Weighted } C = \underline{0.235}$$

Step 2: Determine Weighted C for proposed (improved) conditions using Equation 3-2.

$$\text{Weighted } C = [(2.2)(0.90)+(0.66)(0.15)+(7.52)(0.25)+(7.17)(0.22)] / (17.55)$$

$$\text{Weighted } C = \underline{0.315}$$

English Units

Step 1: Determine Weighted C for existing (unimproved) conditions using Equation 3-2.

$$\text{Weighted } C = \sum (C_x A_x)/A$$
$$=[(22.1)(0.25)+(21.2)(0.22)] / (43.3)$$
$$\text{Weighted } C = \underline{0.235}$$

Step 2: Determine Weighted C for proposed (improved) conditions using Equation 3-2.

$$\text{Weighted } C = [(5.4)(0.90)+(1.6)(0.15)+(18.6)(0.25)+ (17.7)(0.22)] / (43.3)$$

$$\text{Weighted } C = \underline{0.315}$$

A frequency-of-event correction factor, C_f, is sometimes used as a modifier to the Rational formula runoff coefficient. This coefficient is recommended for use by some agencies but is not endorsed by FHWA. The intent of the correction factor is to compensate for the reduced effect of infiltration and other hydrologic abstractions during less frequent, higher intensity storms. The frequency-of-event correction factor is multiplied times the runoff coefficient, C, to produce an adjusted runoff coefficient. Adjustment factors are tabulated by return period below.

$T_r < 25$ years	$C_f = 1.00$
$T_r = 25$ years	$C_f = 1.10$
$T_r = 50$ years	$C_f = 1.20$
$T_r = 100$ years	$C_f = 1.25$

3.2.2.2 Rainfall Intensity

Rainfall intensity, duration, and frequency curves are necessary to use the Rational method. Regional IDF curves are available in most state highway agency manuals and are also available from the National Oceanic and Atmospheric Administration (NOAA). Again, if the IDF curves are not available, they need to be developed.

3.2.2.3 Time of Concentration

There are a number of methods that can be used to estimate time of concentration (t_c), some of which are intended to calculate the flow velocity within individual segments of the flow path (e.g., shallow concentrated flow, open channel flow, etc.). The time of concentration can be calculated as the sum of the travel times within the various consecutive flow segments. For additional discussion on establishing the time of concentration for inlets and drainage systems, see Section 7.2.2 of this manual.

Sheet Flow Travel Time. Sheet flow is the shallow mass of runoff on a planar surface with a uniform depth across the sloping surface. This usually occurs at the headwater of streams over relatively short distances, rarely more than about 130 m (400 ft), and possibly less than 25 m (80 ft). Sheet flow is commonly estimated with a version of the kinematic wave equation, a derivative of Manning's equation, as follows:(6)

$$T_{ti} = \frac{K_u}{I^{0.4}} \left(\frac{n\,L}{\sqrt{S}} \right)^{0.6} \tag{3-3}$$

where:

T_{ti}	=	Sheet flow travel time, min
n	=	Roughness coefficient (see Table 3-2)
L	=	Flow length, m (ft)
I	=	Rainfall intensity, mm/hr (in/hr)
S	=	Surface slope, m/m (ft/ft)
K_u	=	Empirical coefficient equal to 6.92 (0.933 in English units)

Since I depends on t_c and t_c is not initially known, the computation of t_c is an iterative process. An initial estimate of t_c is assumed and used to obtain I from the IDF curve for the locality. The t_c is then computed from Equation 3-3 and used to check the initial value of t_c. If they are not the same, the process is repeated until two successive t_c estimates are the same.(6)

Table 3-2. Manning's Roughness Coefficient (n) for Overland Sheet Flow.[6]	
Surface Description	n
Smooth asphalt	0.011
Smooth concrete	0.012
Ordinary concrete lining	0.013
Good wood	0.014
Brick with cement mortar	0.014
Vitrified clay	0.015
Cast iron	0.015
Corrugated metal pipe	0.024
Cement rubble surface	0.024
Fallow (no residue)	0.05
Cultivated soils	
Residue cover ≤ 20%	0.06
Residue cover > 20%	0.17
Range (natural)	0.13
Grass	
Short grass prairie	0.15
Dense grasses	0.24
Bermuda grass	0.41
Woods*	
Light underbrush	0.40
Dense underbrush	0.80
*When selecting n, consider cover to a height of about 30 mm. This is only part of the plant cover that will obstruct sheet flow.	

Shallow Concentrated Flow Velocity. After short distances of at most 130 m (400 ft), sheet flow tends to concentrate in rills and then gullies of increasing proportions. Such flow is usually referred to as shallow concentrated flow. The velocity of such flow can be estimated using a relationship between velocity and slope as follows [6]:

$$V = K_u \, k \, S_p^{0.5} \tag{3-4}$$

where:

K_u	=	1.0 (3.28 in English units)
V	=	Velocity, m/s (ft/s)
k	=	Intercept coefficient (Table 3-3)
S_p	=	Slope, percent

Table 3-3. Intercept Coefficients for Velocity vs. Slope Relationship of Equation 3-4.[6]	
Land Cover/Flow Regime	k
Forest with heavy ground litter; hay meadow (overland flow)	0.076
Trash fallow or minimum tillage cultivation; contour or strip cropped; woodland (overland flow)	0.152
Short grass pasture (overland flow)	0.213
Cultivated straight row (overland flow)	0.274
Nearly bare and untilled (overland flow); alluvial fans in western mountain regions	0.305
Grassed waterway (shallow concentrated flow)	0.457
Unpaved (shallow concentrated flow)	0.491
Paved area (shallow concentrated flow); small upland gullies	0.619

Open Channel and Pipe Flow Velocity. Flow in gullies empties into channels or pipes. Open channels are assumed to begin where either the blue stream line shows on USGS quadrangle sheets or the channel is visible on aerial photographs. Cross-section geometry and roughness should be obtained for all channel reaches in the watershed. Manning's equation can be used to estimate average flow velocities in pipes and open channels as follows:

$$V = (K_u/n)\ R^{2/3}\ S^{1/2} \quad (3\text{-}5)$$

where:

- n = Roughness coefficient (see Table 3-4)
- V = Velocity, m/s (ft/s)
- R = Hydraulic radius (defined as the flow area divided by the wetted perimeter), m (ft)
- S = Slope, m/m (ft/ft)
- K_u = Units conversion factor equal to 1 (1.49 in English units)

For a circular pipe flowing full, the hydraulic radius is one-fourth of the diameter. For a wide rectangular channel (W > 10 d), the hydraulic radius is approximately equal to the depth. The travel time is then calculated as follows:

$$T_{ti} = L\ /\ (60\ V) \quad (3\text{-}6)$$

where:

- T_{ti} = Travel time for Segment I, min
- L = Flow length for Segment I, m (ft)
- V = Velocity for Segment I, m/s (ft/s)

Example 3-2

Given: *The following flow path characteristics:*

Flow Segment	Length (m)	(ft)	Slope (m/m)(ft/ft)	Segment Description
1 (sheet flow)	68	223	0.005	Bermuda grass
2 (shallow con.)	79	259	0.006	Grassed waterway
3 (flow in conduit)	146	479	0.008	380 mm (15 in) concrete pipe

Find: *Time of concentration, t_c, for the area.*

Solution:

<u>SI Units</u>

Step 1. Calculate time of concentration for each segment, starting at the downstream end, using the 10 - year IDF curve.

<u>Segment 1</u>

Obtain Manning's n roughness coefficient from Table 3-4: $n = 0.011$
Determine the pipe flow velocity from Equation 3-5 assuming full flow for this example
$V = (1.0/0.011)(0.38/4)^{0.67} (0.008)^{0.5} = 1.7 \text{ m/s}$
Determine the travel time from Equation 3-6:
$T_{t3} = L/(60\ V) = 146/[(60)(1.7)] = 1.4 \text{ min}$

<u>Segment 2</u>

Obtain intercept coefficient, k, from Table 3-3:
$k = 0.457$ *and* $K_u = 1.0$
Determine the concentrated flow velocity from Equation 3-4:
$V = K_u\, k\, S_p^{0.5} = (1.0)(0.457)(0.6)^{0.5} = 0.35 \text{ m/s}$
Determine the travel time from Equation 3-6:
$T_{t2} = L/(60\ V) = 79/[(60)(0.35)] = 3.7 \text{ min}$

<u>Segment 3</u>

Obtain Manning's n roughness coefficient from Table 3-2: $n = 0.41$
Determine the sheet flow travel time using Equation 3-3:

$$T_{ti} = (K_u / I^{0.4}) \{[(nL) / (S^{0.5})]^{0.6}\}$$

Since I is being sought and is also in the equation, an iterative approach must be used by estimating a time of concentration (which includes the channel and shallow concentrated flow times computed above) and then reading a rainfall intensity from the appropriate IDF curve. In this example, try a sheet-flow time of concentration of 25 minutes. The time of concentration for the entire watershed is then 25 + 5.1 = 30.1 min (say 30 min). From the IDF curve in Figure 3-1 the intensity is 90 mm/hr. Now use Equation 3-3 to see how good the 25 minute estimate for sheet flow was.

First, solve the equation in terms of I.

T_{t1} = $[6.92/(I)^{0.4}]\ [(0.41)(68)/(0.005)^{0.5}]^{0.6} = (249.8)/\ I^{0.4.}$

Insert 90 mm for I, one gets 41.3 min. Since 41.3 > the assumed 25 min, try the intensity for 41 + 5 = 46 minutes from Figure 3-1 which is 69 mm/hr.

Using 69 mm, one gets 45.9 min. Repeat the process for 46 min + 5 = 51 and a time of 47 min was found. This value is close to the 46 min.

Use 47 minutes for Segment 1.

Step 2. Determine the total travel time by summing the individual travel times:

$t_c = T_{t1} + T_{t2} + T_{t3} = 47.0 + 3.7 + 1.4 = 52.1$ min; use 52 minutes

English Units

Step 1. Calculate time of concentration for each segment, starting at the downstream end, using the 10-year IDF curve.

Segment 1

Obtain Manning's n roughness coefficient from Table 3-4:
$n = 0.011$
Determine the pipe flow velocity from Equation 3-5 (assuming full Flow)
$V = (1.49/0.011)(1.25/4)^{0.67}\ (0.008)^{0.5} = 5.58$ ft/s
Determine the travel time from Equation 3-6:
$T_{t3} = L/(60\ V) = 479/[(60)(5.58)] = 1.4$ min

Segment 2

Obtain intercept coefficient, k, from Table 3-3:
$k = 0.457$ & $K_u = 3.281$
Determine the concentrated flow velocity from Equation 3-4:
$V = K_u\ k\ S_p^{0.5} = (3.281)\ (0.457)(0.6)^{0.5} = 1.16$ ft/s
Determine the travel time from Equation 3-6:
$T_{t2} = L/(60\ V) = 259/[(60)(1.16)] = 3.7$ min

Segment 3

Obtain Manning's n roughness coefficient from Table 3-2: n = 0.41

Determine the sheet flow travel time using Equation 3-3:

$$T_{ti} = (K_u / I^{0.4})\ \{[(nL) / (S^{0.5})]^{0.6}\}$$

Since I is being sought and is also in the equation, an iterative approach must be used by estimating a time of concentration (which includes the channel and shallow concentrated flow times computed above) and then reading a rainfall intensity from the appropriate IDF curve. In this example, try a sheet flow time of concentration of 25 minutes. The time of concentration for the entire watershed is the 25 + 5.1 = 30.1 min *(say 30). From the IDF curve in Figure 3-1 the intensity is 3.4 in/hr. Now use Equation 3-3 to see how good the 25 minute estimate for sheet flow was.*

First, solve the equation in terms of I.

$T_{t1} = [0.933/(I)^{0.4}]\,[(0.41)(223)/(0.005)^{0.5}]^{0.6} = (68.68)/I^{0.4.}$

Insert 3.4 in/hr for I, one gets 42.1 min. Since 42.1> the assumed 25 min, try the intensity for 42 + 5 = 47 minutes from Figure 3-1 which is 2.7 in/hr.

Using 2.7 in/hr., one gets 46.2 min. Repeat the process for 46 + 5 = 51 min and a time of 47 was found. This value is close to the 46.2 min.

Use 47 minutes for Segment 1.

Step 2. Determine the total travel time by summing the individual travel times:

$t_c = T_{t1} + T_{t2} + T_{t3} = 47.0 + 3.7 + 1.4 = 52.1$ *min; use* *52 minutes*

Table 3-4. Typical Range of Manning's Coefficient (n) for Channels and Pipes.	
Conduit Material	Manning's n*
Closed Conduits	
Concrete pipe	0.010 - 0.015
CMP	0.011 - 0.037
Plastic pipe (smooth)	0.009 - 0.015
Plastic pipe (corrugated)	0.018 - 0.025
Pavement/gutter sections	0.012 - 0.016
Small Open Channels	
Concrete	0.011 - 0.015
Rubble or riprap	0.020 - 0.035
Vegetation	0.020 - 0.150
Bare Soil	0.016 - 0.025
Rock Cut	0.025 - 0.045
Natural channels (minor streams, top width at flood stage <30 m (100 ft))	
Fairly regular section	0.025 - 0.050
Irregular section with pools	0.040 - 0.150
*Lower values are usually for well-constructed and maintained (smoother) pipes and channels	

Example 3-3

***Given**: Land use conditions from Example 3-1 and the following times of concentration:*

	Time of concentration t_c (min)	*Weighted C (from Example 3-1)*
Existing condition (unimproved)	*88*	*0.235*
Proposed condition (improved)	*66*	*0.315*

Area = 17.55 ha (43.36 acres)

***Find:** The 10-year peak flow using the Rational Formula and the IDF Curve shown in Figure 3-1.*

Solution:

<u>SI Units</u>

Step 1. Determine rainfall intensity, I, from the 10-year IDF curve for each time of concentration.

Rainfall intensity, I

Existing condition (unimproved) 48 mm/hr

Proposed condition (improved) 58 mm/hr

Step 2. Determine peak flow rate, Q.

Existing condition (unimproved):

$$Q = CIA / K_u = (0.235)(48)(17.55)/360 = \underline{0.55\ m^3}/s$$

Proposed condition (improved):

$$Q = CIA / K_u = (0.315)(58)(17.55)/360 = \underline{0.89\ m^3}/s$$

<u>English Units</u>

Step 1. Determine rainfall intensity, I, from the 10-year IDF curve for each time of concentration.

Rainfall intensity, I

Existing condition (unimproved) 1.9 in/hr

Proposed condition (improved) 2.3 in/hr

Step 2. Determine peak flow rate, Q.

Existing condition (unimproved):

$$Q = CIA / K_u = (0.235)(1.9)(43.3)/1 = \underline{19.3\ ft^3}/s$$

Proposed condition (improved):

$$Q = CIA / K_u = (0.315)(2.3)(43.3)/1 = \underline{31.4\ ft^3}/s$$

Reference 6 contains additional information on the Rational method.