STABILITY, BEARING CAPACITY, AND REINFORCEMENT COMPUTATIONS, EXAMPLES

N-1. EXAMPLE 1. Analyze the wall shown below for stability and bearing capacity. Find the reinforcement required at critical sections. Load Case R1.

Given:

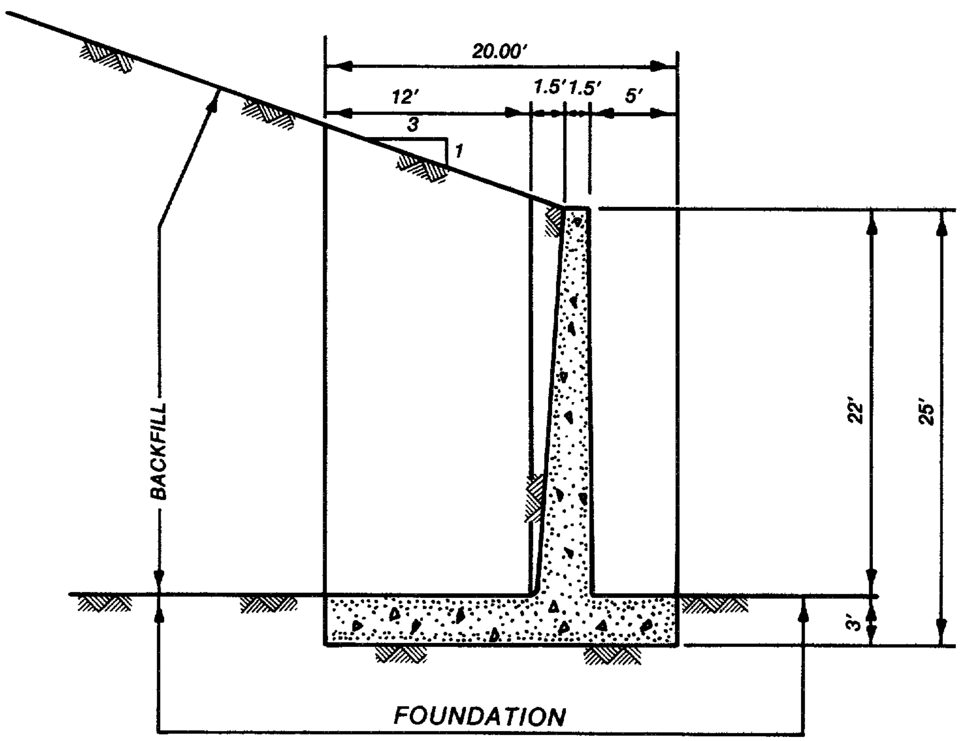
Backfill: γ = 0.120 kcf, ϕ = 35°, c = 0

Foundation: γ = 0.135 kcf, ϕ = 40°, c = 0

Reinforced concrete: γ = 0.150 kcf, f'_c = 4 ksi, f_y = 48 ksi

All concrete cover = 4.5 in. (to center of gravity of steel)

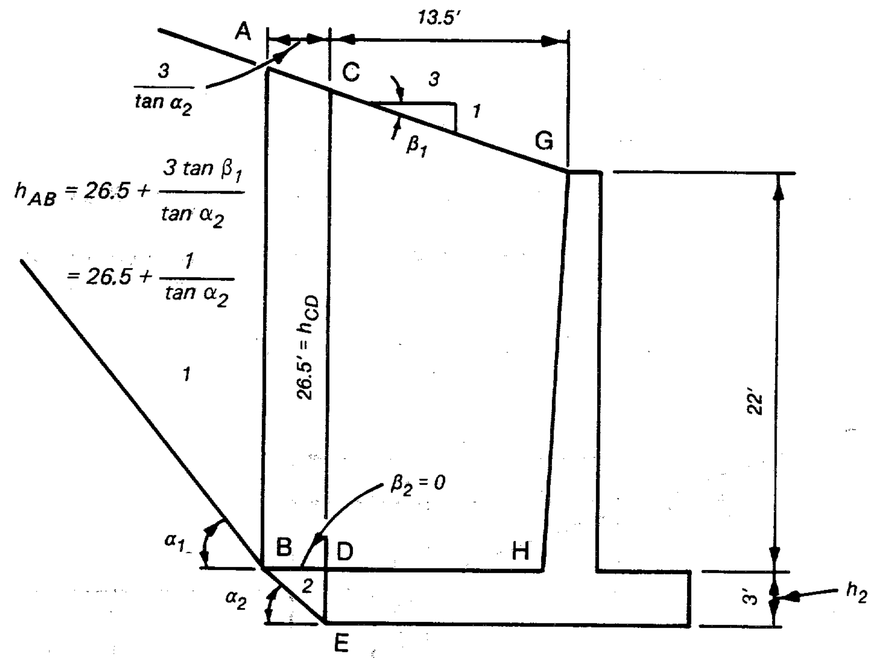
Load factor = 1.9, ϕ = 0.9



a. <u>Find critical slip-plane angles--driving side</u>.

(1) Foundation (see Appendix G):

$\phi = 40°$, SMF = 2/3 , $\phi_d = \tan^{1}(2/3 \tan \phi) = 29°$



The increased unit weight in the bottom layer (layer 2) is calculated as

$$\gamma' = \frac{2\gamma_1 h_1}{h_2} + \gamma_2 + \frac{2\gamma_1 (\tan \beta_1 - \tan \beta_2)}{\tan \alpha_2 - \tan \beta_2} \qquad \text{[G-25]}$$

$$\gamma' = \frac{2(0.12)(26.5)}{3} + 0.135 + \frac{2(0.12)(1/3)}{\tan \alpha_2} = 2.255 + \frac{0.08}{\tan \alpha_2}$$

$$V_\alpha = \frac{\gamma_1 h_2^2 (\tan \beta_1 - \tan \beta_2)}{2\left(\tan \alpha_2 - \tan \beta_2\right)^2} \qquad \text{[G-26]}$$

$$V_\alpha = \frac{0.12(3)^2(1/3)}{2\tan^2\alpha_2} = \frac{0.18}{\tan^2\alpha_2}$$

Let $\alpha_2 = 59.5° = 45 + \frac{\phi_d}{2}$,

$$\gamma' = 2.255 + \frac{0.08}{1.697663} = 2.302 ,$$

$$V_\alpha = \frac{0.18}{(1.697663)^2} = 0.0625$$

From Equation G-27 (omitting the c term)

$$A' = \tan\phi_d - \frac{2V_\alpha\left(1 + \tan^2\phi_d\right)}{\gamma' h_2^2}$$

$$A' = 0.554309 - \frac{2(0.0625)(1.307259)}{2.302(3)^2} = 0.546422$$

From Equation G-28 (omitting the c term)

$$c_1' = \left[2\tan^2\phi_d - \frac{4V_\alpha \tan\beta_2\left(1 + \tan^2\phi_d\right)}{\gamma' h_2^2}\right] \div A'$$

$$c_1' = 2(0.554309)^2 \div 0.546422 = 1.12462$$

From Equation G-29 (omitting the c term)

$$c_2' = \frac{\tan \phi_d \left(1 - \tan \phi_d \tan \beta_2\right) - \tan \beta_2 + \dfrac{2V_\alpha \tan^2 \beta_2 \left(1 + \tan^2 \phi_d\right)}{\gamma' h_2^2}}{A'}$$

$$c_2' = 0.554309 \div 0.546422 = 1.01443$$

$$\alpha_2 = \tan^{-1}\left(\frac{c_1' + \sqrt{c_1'^2 + 4c_2'}}{2}\right) = 59.77° \neq 59.5° \qquad \text{[G-30]}$$

Let $\alpha_2 = 59.73°$, $\gamma' = 2.302$, $V_\alpha = 0.061$

$$A' = 0.554309 - \frac{2(0.061)(1.307259)}{2.302(3)^2} = 0.546611$$

$$c_1' = 2(0.554309)^2 \div 0.546611 = 1.124231$$

$$c_2' = 0.554309 \div 0.546611 = 1.014083$$

$\alpha_2 = 59.76°$ 59.73° (close enough)

$\alpha_2 = \underline{\underline{59.76°}}$

(2) Backfill:

$$h_{AB} = 26.5 + \frac{3 \tan \beta_1}{\tan \alpha_2} = \underline{\underline{27.083 \text{ ft}}}, \quad \phi = 35°$$

$$\tan \phi_d = \tan^{-1} (2/3 \tan \phi) = 25°$$

$$c_1 = 2 \tan \phi_d = 2(0.466308) = 0.932616 \qquad \text{[3-26]}$$

$$c_2 = 1 - \tan \phi_d \tan \beta_1 - \frac{\tan \beta_1}{\tan \phi_d} \qquad [3\text{-}27]$$

$$c_2 = 1 - 0.466308\left(\frac{1}{3}\right) - \frac{\frac{1}{3}}{0.466308} = 0.129729$$

$$\alpha_1 = \underline{\underline{46.55°}} \qquad [3\text{-}25]$$

b. Earth pressure coefficients (see Appendix H).

(1) Backfill:

$$K_1 = \frac{1 - \tan \phi_d \cot \alpha_1}{1 + \tan \phi_d \tan \alpha_1} \cdot \frac{\tan \alpha_1}{\tan \alpha_1 - \tan \beta_1}$$

$$K_1 = \frac{1 - 0.466308 \times 0.947307}{1 + 0.466308 \times 1.055624} \cdot \frac{1.055624}{1.055624 - \frac{1}{3}} = \underline{\underline{0.5468}}$$

(2) Foundation:

$$K = \frac{1 - \tan \phi_d \cot \alpha_2}{1 + \tan \phi_d \tan \alpha_2} = \frac{1 - 0.554309 \times 0.582949}{1 + 0.554309 \times 1.715416} = \underline{\underline{0.3470}}$$

$$K_v = K \tan \alpha_2 = \underline{\underline{0.5952}}$$

c. Lateral pressure and forces on Surfaces AB and DE.

$$P_{AB} = \frac{1}{2} K_1 \gamma_1 h_{AB}^2 = \frac{1}{2} (0.5468)(0.12)(27.083)^2 = \underline{\underline{24.06 \text{ kips}}}$$

Surface DE:

$$P_{DE} = \frac{1}{2} K\gamma_2 h_2^2 + K_v V$$

$$V = \frac{1}{2} (h_{AB} + h_{CD})(\gamma_1)\left(\frac{3}{\tan \alpha_2}\right)$$

$$V = \frac{1}{2} (27.083 + 26.5)(0.12)\left(\frac{3}{1.715416}\right) = 5.623 \text{ kips}$$

$$P_{DE} = \frac{1}{2} (0.347)(0.135)(3)^2 + 0.5952(5.623) = 3.5576 \text{ kips}$$

$$p_B = K_1\gamma_1 h_{AB} = 0.5468(0.12)(27.083) = 1.7771 \text{ ksf}$$

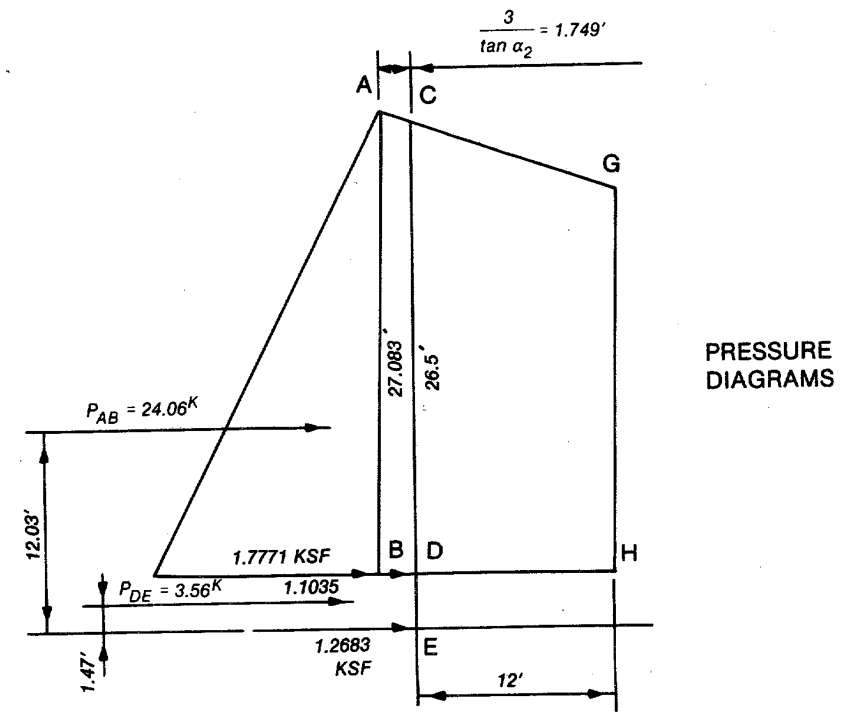
$$p_D = K\gamma_1 h_{CD} = 0.3470(0.12)(26.5) = 1.1035 \text{ ksf}$$

$$P_{DE} = \left(\frac{p_D + p_E}{2}\right) h_2$$

$$p_E = \frac{2P_{DE}}{h_2} - p_D$$

$$p_E = \frac{2(3.5576)}{3} - 1.1035$$

$$p_E = 1.2683 \text{ ksf}$$

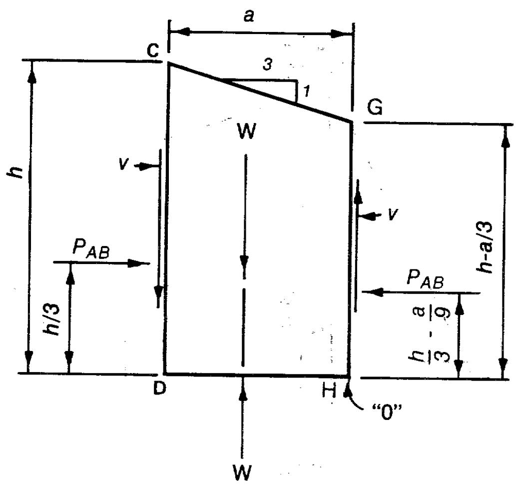


d. <u>Shear force on structural wedge (paragraphs 4-8c, 9-7a)</u>.

(1) The horizontal force P_{AB} will be considered to act on any vertical plane in the soil that lies on Surface AB, or on any surface to the right of Surface AB, such as surfaces CD or GH.

(2) In order for the body of soil lying between CD and GH to be in equilibrium, a small vertical shear will be assumed to act on vertical surfaces along with the horizontal force P_{AB} .

(3) The value of this shear force is found from a free body of any block of soil as shown on the following page:



$$\Sigma M_o = 0 = P_{AB}\left[\frac{h}{3} - \left(\frac{h}{3} - \frac{a}{9}\right)\right] - va$$

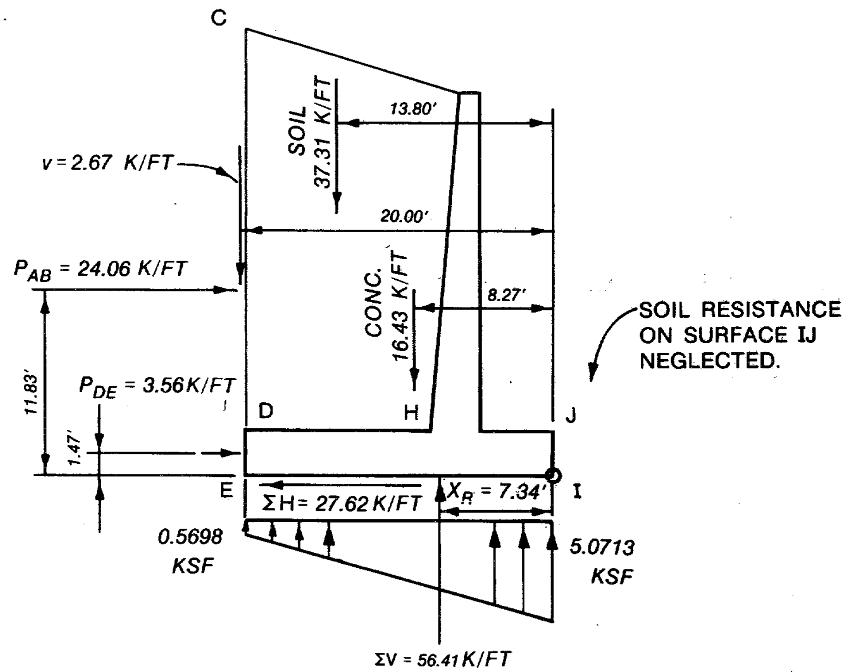
$$va = P_{AB}\left(\frac{a}{9}\right), \quad v = \frac{P_{AB}\tan\beta}{3} = \frac{P_{AB}}{9} = \frac{24.06}{9} = 2.673 \text{ kips}$$

e. Weight and center of gravity of structural wedge.

1.5 ft x 22 ft x 0.15 kcf =	4.95	x	5.75 ft =	28.46 kips		
$\frac{1}{2}$ x 1.5 ^Cft x 22 ft x 0.15 kcf =	2.48	x	7.00 ft =	17.36 kips		
20 ft x 3 ft x 0.15 kcf =	9.00	x	10.00 ft =	90.00 kips		
Concrete =	16.43 k/ft				135.82 ÷ 16.43 = 8.27 ft = $\bar{x}$	

$\frac{1}{2}$ x 13.5 ft x 4.5 ft x 0.12 kcf =	3.65	x	15.50 ft =	56.58	
12 ft x 22 ft x 0.12 kcf =	31.68	x	14.00 ft =	443.52	
$\frac{1}{2}$ x 1.5 ft x 22 ft x 0.12 kcf =	1.98	x	7.50 ft =	14.85	
Soil =	37.31 k/ft				514.95 ÷ 37.31 = 13.80 ft = $\bar{x}$

f. Overturning stability (paragraph 4-8).



$2.67 \text{ kips} \times 20.00 \text{ ft} = 53.400 \text{ ft-kips}$
$37.31 \text{ kips} \times 13.80 \text{ ft} = 514.878 \text{ ft-kips}$
$\underline{16.43} \text{ kips} \times 8.27 \text{ ft} = 135.876 \text{ ft-kips}$
$56.41 = \Sigma V$

$-24.06 \times 11.83 = -284.630$
$\underline{-3.56} \times 1.47 = \underline{-5.233}$
$-27.62 = \Sigma H \qquad 414.291 = \Sigma M_I$

$$x_R = \frac{414.291}{56.41} = 7.34 \text{ ft}$$

One hundred percent of base is in compression, overturning stability requirement is satisfied.

g. Sliding stability analysis (paragraph 4-15).

$N' = \Sigma V = 56.41$ kips, $T = \Sigma H = 27.62$ kips

$$T \leq \frac{N' \tan \phi + cL}{FS} \qquad [4\text{-}12]$$

Using the minimum FS of 1.5 from Table 4-1 yields

$$27.62 \leqq \frac{56.41 \tan 40° + 0\ (20)}{1.5}$$

$$27.62 \leqq 31.56$$

Sliding stability requirement is satisfied.

h. Check bearing capacity (Chapter 5).

$$\delta = \tan^{-1}\left(\frac{27.62}{56.41}\right) = 26° = 0.4538 \text{ rad} \qquad \text{(Figure 5-1(a))}$$

$$e = \frac{B}{2} - x_R = \frac{20}{2} - 7.34 = 2.66 \text{ ft}$$

$$\overline{B} = B - 2e = 20 - 2(2.66) = 14.68 \text{ ft}$$

$$q_o = \gamma_D = 0.135(3) = 0.405 \text{ ksf} \qquad [5\text{-}8a]$$

$$\xi_{qi} = \left(1 - \frac{\delta}{90}\right)^2 = 0.5057 \qquad [5\text{-}5a]$$

$$\xi_{\gamma i} = \left(1 - \frac{\delta}{\phi}\right)^2 = \left(1 - \frac{26}{40}\right)^2 = 0.1225 \qquad [5\text{-}5b]$$

$$N_q = 64.20\ , \quad N_\gamma = 93.69 \qquad \text{(Table 5-1)}$$

Using Equation 5-2 yields

$$Q = \overline{B}\left(\xi_{qi} q_o N_q + \frac{\xi_{\gamma i}\ \overline{B} \gamma N_\gamma}{2}\right)$$

$$Q = 14.68\left[0.5057(0.405)(64.20) + \frac{0.1225(14.68)(0.135)(93.69)}{2}\right]$$

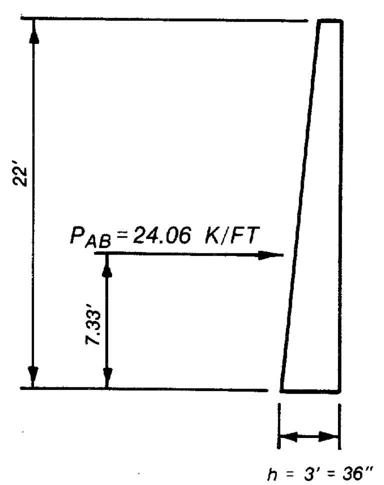
$$Q = 359.97 \text{ kips}$$

For this problem $\Sigma V = N'$. The factor of safety is calculated as

$$FS = \frac{Q}{N'} = \frac{359.97}{56.41} = 6.38 > 3.00 \qquad [5\text{-}1]$$

Bearing capacity requirement is satisfied (Table 4-1).

i. <u>Reinforcement at base of stem (Chapter 9)</u>. Neglect vertical shear component v .



$$M = 24.06(7.33 \text{ ft}) = 176.36 \text{ k-ft}$$

$$k_u = 1 - \sqrt{1 - \frac{\frac{M_u}{\phi}}{0.425 f'_c bd^2}} , \quad \text{(Figure 9-2)}$$

$$\frac{M_u}{\phi} = \frac{1.9(176.36)(12)}{0.90} = 4{,}468 \text{ k-in.}$$

$$b = 12 \text{ in.}, \quad d = h - 4.5 = 31.5 \text{ in.}$$

$$0.425 f'_c bd^2 = 0.425(4)(12)(31.5)^2 = 20{,}241.9$$

$$k_u = 1 - \sqrt{1 - \frac{4{,}468}{20{,}241.9}} = 0.1174$$

$$C_u = T_u = 0.85 f'_c k_u bd$$

$$C_u = T_u = 0.85(4)(0.1174)(12)(31.5)$$

$$= 150.88 \text{ kips}$$

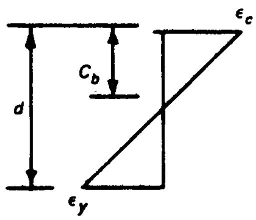
$$A_s = \frac{T_u}{f_y} = \frac{150.88}{48} = \underline{\underline{3.14 \text{ in.}^2/\text{ft}}}$$

Check if ductility requirement of $\rho_{min} \leq \rho \leq \rho_{max}$ is satisfied:

$$\rho_{max} = \lambda \rho_b \;, \quad \lambda = 0.25 \quad \text{(paragraph 9-8b(3))}$$

$$\rho_{min} = 200/f_y = 200/48{,}000 = 0.00417 \text{ (paragraph 9-8b(4), from ACI 318)}$$

The reinforcement ratio ρ_b in the balanced condition may be obtained by applying equilibrium and compatibility conditions. From the linear strain condition shown below:



$$\frac{c_b}{d} = \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_y} = \frac{0.003}{0.003 + \dfrac{f_y}{29{,}000 \text{ ksi}}}$$

The compressive force C is

$$C = 0.85f'_c\beta_1 bc_b$$

The tensile force is

$$T = f_y A_{sb} = \rho_b bdf_y$$

Equating C to T yields

$$0.85f'_c b\beta_1 c_b = \rho_b bdf_y$$

$$\rho_b = \frac{0.85f'_c}{f_y} \beta_1 \frac{c_b}{d} = \frac{0.85f'_c}{f_y} \beta_1 \left(\frac{\varepsilon_c}{\varepsilon_c + \dfrac{f_y}{29{,}000 \text{ ksi}}} \right)$$

$\beta_1 = 0.85$, $\varepsilon_c = 0.003$ (for nonhydraulic structure, paragraph 9-8e)

$$\rho_b = \frac{0.85(4)}{48}(0.85)\left(\frac{0.003}{0.003 + \frac{48}{29,000}}\right) = 0.03880$$

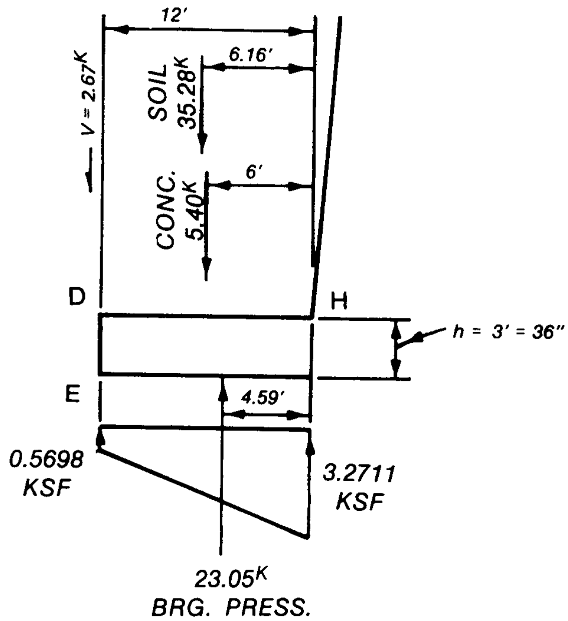
$$\rho_{max} = 0.25(0.03880) = 0.009700$$

$$\rho = \frac{A_s}{bd} = \frac{3.14}{12(31.5)} = 0.008307$$

$$0.00417 < \rho = 0.008307 < 0.009700$$

Ductility is adequate.

j. <u>Reinforcement in heel at face of stem.</u>



$$\begin{aligned} 2.67 \times 12.00 &= 32.04 \\ 35.28 \times 6.16 &= 217.32 \\ 5.40 \times 6.00 &= 32.40 \\ -23.05 \times 4.59 &= \underline{-105.80} \\ M &= 175.96 \text{ k-ft} \end{aligned}$$

From Figure 9-2

$$\frac{M_u}{\phi} = \frac{1.9(175.96)(12)}{0.9} = 4,458 \text{ k-in.}$$

$$b = 12 \text{ in.}, \quad d = 36 - 4.5 = 31.5 \text{ in.}$$

$$0.425 f'_c bd^2 = 20,241.9$$

$$k_u = 1 - \sqrt{1 - \frac{4,458}{20,241.9}} = 0.1170$$

$$C_u = T_u = 0.85(4)(0.1170)(12)(31.5)$$

$$= 150.37 \text{ kips}$$

$$A_s = \frac{150.37}{48} = \underline{\underline{3.13 \text{ in.}^2/\text{ft}}}$$

Check ductility:

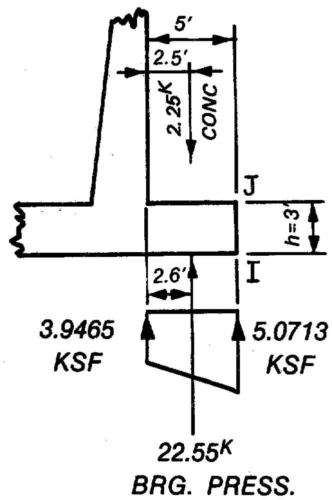
$$\rho_{max} = 0.009700 , \quad \rho_{min} = 0.00417$$

$$\rho = \frac{A_s}{bd} = \frac{3.13}{12(31.5)} = 0.008280$$

$$0.00417 < \rho = 0.008280 < 0.009700$$

Ductility is adequate.

k. Reinforcement in toe at face of stem.



$$22.55 \times 2.60 = 58.63$$
$$-2.25 \times 2.50 = \underline{-5.625}$$
$$M = 53.01 \text{ k-ft}$$

From Figure 9-2

$$\frac{M_u}{\phi} = \frac{1.9(53.01)(12)}{0.9} = 1,342.8 \text{ k-in.}$$

$$b = 12 \text{ in.}, \quad d = 36 \text{ in.} - 4.5 = 31.5 \text{ in.}$$

$$0.425f'_c bd^2 = 20,241.9$$

$$k_u = 1 - \sqrt{1 - \frac{1,342.8}{20,241.9}} = 0.0337$$

$$C_u = T_u = 0.85(4)(0.0337)(12)(31.5)$$
$$= 43.31 \text{ kips}$$

$$A_s = \frac{43.31}{48} = \underline{\underline{0.90 \text{ in.}^2/\text{ft}}}$$

Check ductility:

$$\rho_{max} = 0.02910 , \quad \rho_{min} = 0.00417$$

$$\rho = \frac{A_s}{bd} = \frac{0.90}{12(31.5)} = 0.002381$$

$$0.00417 < \rho = 0.002381 < 0.009700$$

Ductility is adequate.

l. Shear check (paragraph 9-8f). The shear capacity of the concrete will be checked at a distance d from the base of the stem according to ACI 318.

$$d_v = 31.5 - \frac{1.5}{22} d_v , \quad 1.06818\, d_v = 31.5$$

d_v = 29.5 in. = 2.46 ft (member depth for shear at distance d_v above base)

Since the shear has a quadratic variation, the shear at distance d_v can be calculated as shown below.

$$V = \frac{(22 - 2.46)^2}{(22)^2} (24.06) = 18.98 \text{ kips}$$

$$V_u = 1.9(18.98) = 36.06 \text{ kips}$$

$$\phi V_c = (0.85)(2) \sqrt{f'_c}\, bd_v \qquad \text{(from ACI 318)}$$

$$\phi V_c = (0.85)(2)\sqrt{4{,}000}(12)(29.5) = 38{,}061 \text{ lb} > V_u$$

Check the shear capacity of the heel at the base of the stem:

$$V = 2.67 + 5.40 + 35.28 - 23.05 = 20.3 \text{ kips}$$

$$V_u = 1.9\ (20.3) = 38.57 \text{ kips}$$

$$\phi V_c = (0.85)(2) \sqrt{f'_c}\, bd_v$$

$$\phi V_c = (0.85)(2)\sqrt{4{,}000}\ (12)(31.5) = 40{,}642 \text{ lb} > V_u$$